

MA1115 Exam # 3

June 6th, 2007

Name \_\_\_\_\_

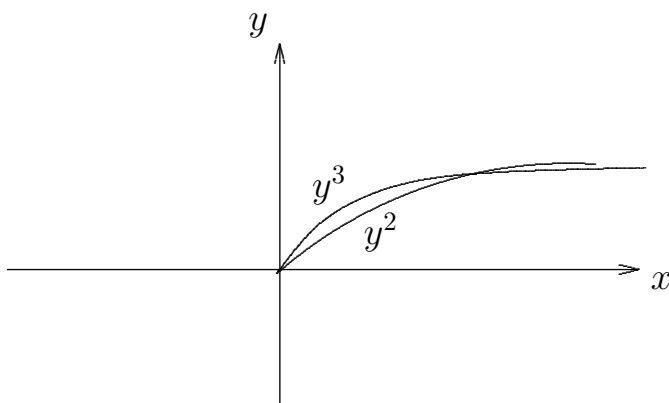
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Show all necessary work in each problem to receive full credit, each problem is worth 20 points. In working the following problems, you may only use your note card and calculator if necessary. Good luck and enjoy!

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1. (20 points) Find the volume under the surface  $z = 2x + 3y^2$  and above the region bounded by  $x = y^2$  and  $x = y^3$

Solution:



$$\int \int_S 2x + 3y^2 dA = \int_0^1 \int_{y^3}^{y^2} 2x + 3y^2 dx dy = \int_0^1 \left[ x^2 + 3xy^2 \right]_{y^3}^{y^2} dy = \int_0^1 y^4 + 3y^4 - y^6 - 3y^5 dy = \frac{11}{70}$$

2. (20 points) Evaluate the iterated integral by converting to polar coordinates

$$\int_0^3 \int_0^{\sqrt{9-y^2}} e^{\sqrt{x^2+y^2}} dx dy$$

Solution:

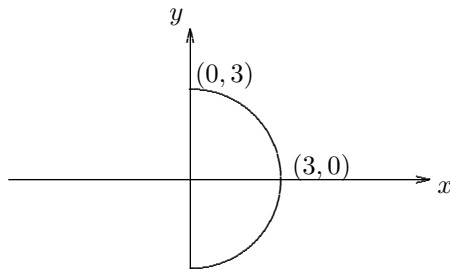
$$\int_0^3 \int_0^{\sqrt{9-y^2}} e^{\sqrt{x^2+y^2}} dx dy = \int_0^{\frac{\pi}{2}} \int_0^3 e^r r dr d\theta = \int_0^{\frac{\pi}{2}} d\theta \int_0^3 e^r r dr$$

using integration by parts for  $\int_0^3 e^r r dr$  we have:

$$u = r \rightarrow du = dr$$

$$dv = e^r dr \rightarrow v = e^r \text{ and so}$$

$$\int_0^{\frac{\pi}{2}} d\theta \int_0^3 e^r r dr = \left(\frac{\pi}{2}\right) \left( r e^r \Big|_0^3 - \int_0^3 e^r dr \right) = \left(\frac{\pi}{2}\right) \left( (3e^3 - 0) - (e^3 - e^0) \right) = \left(\frac{\pi}{2}\right) (2e^3 + 1)$$



**3. (20 points)** (a) Set up an integral that will evaluate  $\int \int \int_E e^{x^2+y^2+z^2} dV$ , where  $E$  lies in the first octant between the spheres  $x^2 + y^2 + z^2 = 9$  and  $x^2 + y^2 + z^2 = 25$  (hint: you might want to use spherical coordinates). (b) What method of integration would you use to solve the integral?

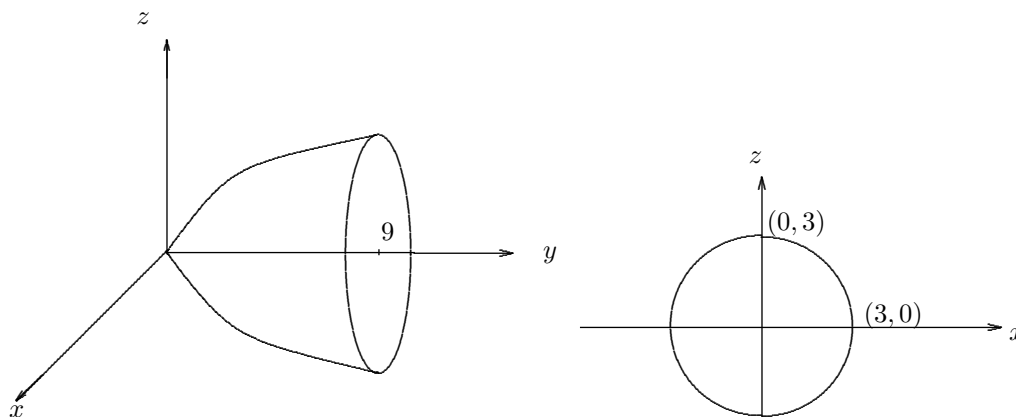
Solution: (a)

$$\int_0^{\pi/2} \int_0^{\pi/2} \int_3^5 e^{\rho^2} \rho \sin \phi \, d\rho d\theta d\phi$$

(b) use integration by parts twice to solve the integral with respect to  $\rho$ , and just integration for the other two integrals.

**4. (20 points)** Set up an integral to find the volume of the solid enclosed by the paraboloid  $y = x^2 + z^2$  and the plane  $y = 9$ .

Solution:



rectangular:

$$\int_{-3}^3 \int_{-\sqrt{9-z^2}}^{\sqrt{9-z^2}} \int_{x^2+z^2}^9 dy dx dz$$

cylindrical:

$$\int_0^{2\pi} \int_0^3 \int_{r^2}^9 r dr d\theta$$

polar:

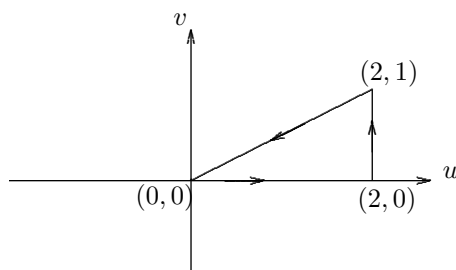
$$\int_0^{2\pi} \int_0^3 (9 - r^2) r dr d\theta$$

rectangular:

$$\int_{-3}^3 \int_{-\sqrt{9-z^2}}^{\sqrt{9-z^2}} 9 - (x^2 + z^2) dx dz$$

**5. (20 points)** Let  $S$  be the triangle in the  $u - v$  plane with vertices  $(0, 0)$ ,  $(2, 0)$  and  $(2, 1)$ . Find the image of  $S$  under the transformation  $x = v$  and  $y = u^2 + v^2$ .

Solution: Note that:



$$S_1 : v = 0, 0 \leq u \leq 2$$

$$x = 0, y = u^2$$

$$S_2 : u = 2, 0 \leq v \leq 1$$

$$x = v, y = 4 + v^2 = 4 + x^2, \text{ with } 0 \leq x \leq 1$$

$$S_3 : v = \frac{u}{2}, 0 \leq u \leq 2$$

$$x = \frac{u}{2}, y = \frac{5u^2}{4} \rightarrow y = 5x^2 \text{ with } 0 \leq x \leq 1$$

